

1) a) $A + A^T = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$ [4]

b) $AB = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & -2 \\ -1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 11 \\ -5 & 2 & 7 \end{bmatrix}$ [6]

c) $A^{-1} = \frac{1}{|A|} \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix} \left(= \begin{bmatrix} 1/5 & -3/5 \\ -1/5 & 2/5 \end{bmatrix} \right)$ [5]

2) Using an augmented matrix,

$$\begin{bmatrix} 6 & 4 & | & 2 \\ 3 & -5 & | & -34 \end{bmatrix} \xrightarrow{r1 \div 2} \begin{bmatrix} 3 & 2 & | & 1 \\ 3 & -5 & | & -34 \end{bmatrix} \xrightarrow{r2 - r1} \begin{bmatrix} 3 & 2 & | & 1 \\ 0 & -7 & | & -35 \end{bmatrix} \xrightarrow{r2 \div (-7)} \begin{bmatrix} 3 & 2 & | & 1 \\ 0 & 1 & | & 5 \end{bmatrix}$$

$$\xrightarrow{r1 - 2r2} \begin{bmatrix} 3 & 0 & | & -9 \\ 0 & 1 & | & 5 \end{bmatrix} \xrightarrow{r1 \div 3} \begin{bmatrix} 1 & 0 & | & -3 \\ 0 & 1 & | & 5 \end{bmatrix} \therefore \text{Solution is } \underline{x = -3, y = 5}$$
 [12]

3) $\int_0^2 \int_0^x 4x^2 - 3y \, dy \, dx = \int_0^2 \left(4x^2 y - \frac{3}{2} y^2 \right) \Big|_{y=0}^{y=x} dx = \int_0^2 4x^3 - \frac{3}{2} x^2 \, dx$

$$= \left(x^4 - \frac{1}{2} x^3 \right) \Big|_0^2 = 16 - 4 = \underline{\underline{12}}$$
 [13]

4) $f(x, y) = 8x - 12y^2 - xy^3$

a) $\therefore f_x = 8 - y^3$ ① $f_y = -24y - 3xy^2$ ②

$f_{xx} = 0$ $f_{yy} = -24 - 6xy$

$f_{xy} = -3y^2$ [10]

b) Have critical points where $f_x = f_y = 0$

$f_x = 8 - y^3 = 0$ where $y = 2$

Then $f_y = -24y - 3xy^2 = -3y(8 + xy) = -6(8 + 2x) = 0$

$\therefore x = -4$

$\therefore$ Have one critical point at $\underline{(-4, 2)}$ [5]

Consider $D = f_{xx} f_{yy} - [f_{xy}]^2 = 0 - (9y^4)$

At $(-4, 2)$, $D < 0 \therefore$ This is a saddle point. [5]

$$[B1] a) \rho_T(10, 32) \approx \frac{\rho(12, 32) - \rho(10, 32)}{12 - 10} = \frac{24.27 - 24.62}{2} = -0.175$$

$$\text{or } \rho_T(10, 32) \approx \frac{\rho(10, 32) - \rho(8, 32)}{10 - 8} = \frac{24.62 - 24.93}{2} = -0.155$$

Take the average : Estimate $\rho_T(10, 32) = \underline{\underline{-0.165 \text{ units}/^\circ\text{C}}}$ [7]

Interpretation : At 10°C and salinity of 32 parts per thousand, the instantaneous rate of decrease of density with temperature (at constant salinity) is $0.165 \text{ units}/^\circ\text{C}$. [3]

b) Similarly we can estimate

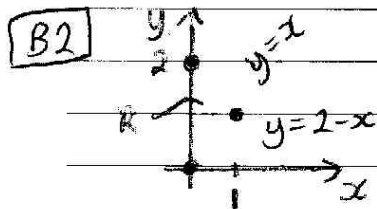
$$\rho_S(10, 32) \approx \frac{\rho(10, 33) - \rho(10, 32)}{33 - 32} = \frac{25.42 - 24.62}{1} = 0.8$$

$$\text{or } \rho_S(10, 32) \approx \frac{\rho(10, 32) - \rho(10, 31)}{32 - 31} = \frac{24.62 - 23.85}{1} = 0.77$$

} average
0.785

[4]

$$\text{Then } L(T, S) = \rho(10, 32) + \rho_T(10, 32)(T - 10) + \rho_S(10, 32)(S - 32) \\ = \underline{\underline{24.62 - 0.165(T - 10) + 0.785(S - 32)}} \quad [6]$$



$$V = \iint_R e^{x+y} dA \quad R = \{(x, y) : x \leq y \leq 2-x, 0 \leq x \leq 1\}$$

$$\therefore V = \int_0^1 \int_x^{2-x} e^{x+y} dy dx$$

$$= \int_0^1 (e^{x+y}) \Big|_{y=x}^{y=2-x} dx = \int_0^1 (e^2 - e^{2x}) dx$$

$$= \left(x e^2 - \frac{1}{2} e^{2x} \right) \Big|_0^1 = e^2 - \frac{1}{2} e^2 + \frac{1}{2} = \underline{\underline{\frac{1}{2} (e^2 + 1)}} \quad [20]$$

B3 Let x be number of fish of species A

y " " " " B

z " " " " C

We have $x + y + z = 100$

$4x + y + 0z = 135$

$2x + y + 5z = 220$

[10]

As an augmented matrix,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 4 & 1 & 0 & 135 \\ 2 & 1 & 5 & 220 \end{array} \right] \xrightarrow[r3-2r1]{r2-4r1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -3 & -4 & -265 \\ 0 & -1 & 3 & 20 \end{array} \right] \xrightarrow[r2, r3]{\text{SWAP}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -1 & 3 & 20 \\ 0 & 3 & 4 & 265 \end{array} \right]$$

$$\xrightarrow{r3+3r2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -1 & 3 & 20 \\ 0 & 0 & 13 & 325 \end{array} \right] \xrightarrow{r3/13} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -1 & 3 & 20 \\ 0 & 0 & 1 & 25 \end{array} \right] \xrightarrow[r2-3r3]{r1-r3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 75 \\ 0 & -1 & 0 & -55 \\ 0 & 0 & 1 & 25 \end{array} \right]$$

$$\xrightarrow[r2 \rightarrow -r2]{r1+r2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 55 \\ 0 & 0 & 1 & 25 \end{array} \right]$$

$\therefore$ Have 20 fish of species A

55 " " " B

25 " " " C.

[10]